**MATHEMATICS IN AND OUT OF SCHOOL CONFERENCE**
**REPORT TO THE NATIONAL SCIENCE FOUNDATION**

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# I. THE CONFERENCE

The Mathematics In and Out of School conference was held on October 21-23, 1999. The conference was sponsored by the PRIMES project at The Institute for Research on Learning.

The meeting brought together an interdisciplinary and diverse group of researchers who have studied mathematics learning and use in school, everyday and professional settings. The group represented 30 researchers and students from education, cognitive science, developmental psychology, anthropology, mathematics, linguistics, and computer science. The researchers have worked nationwide and internationally. Several researchers have studied mathematics learning in school and have been leaders in developing standards, curriculum approaches and research on school mathematics K-graduate school. A second group has studied mathematics from a cultural perspective, coming to understand ways of being mathematical in the lives of various peoples of the world. A third group has studied mathematics at work, coming to understand how line workers, salespersons, professionals and scientists use mathematics to accomplish their work (see appendix A for a list of participants).

The following questions framed the meeting:

1. *What are the mathematical demands in the settings studied? How do we see people learning mathematics? In what ways do people move beyond simple arithmetic? How do people develop and use problem solving, representations, and algorithms in the setting? Who does what kind of mathematical work?*

2. *What is the nature of mathematical communication? How is mathematics negotiated and practiced? How is mathematics embedded in local practices? Is there teaching? If so, what does teaching look like?*

3. *How do mathematical practices in work and everyday settings correspond to school mathematics practices? Do these representations align with the representations in school mathematics?*

4. *What are the roles for K-12, college level and/or work place in the mathematics learning process?*

5. *How can we apply what we know about mathematics learning in school, everyday life, and the work place to school-to-career? Is a blueprint for learning K-adulthood possible? What would be the next steps and who will need to come to the planning table?*

The researchers shared work, discussed issues, and synthesized results of their research and the research of others with special regard for the issues related to more successful school-to-career mathematics capability. The common goal was a better understanding of how linkages can be built between children's learning, mathematics in everyday and work settings, with concern for the content and approach of school mathematics and work place training. The researchers shared insights and results, identified issues, and determined areas and criteria for further study that might truly enable a blueprint for smoother transitions from the school mathematics experience to the needs of those in the workplace.

The following sections contain the recommendations of the group for next steps that must be taken in research to better understand mathematics teaching and learning in school, work and across the life span. The research findings presented at the conference demonstrate that a major program of research and development could be organized that would provide the basis for reshaping mathematics education. This reshaping could fulfill goals for universalizing meaningful mathematics learning in the United States, as envisioned in the *Standards* of the National Council of Teachers of Mathematics and other progressive proposals for mathematics education reform. Recent research and theory have enabled members of the research community to ready themselves for the needed, more coordinated research steps. Activities such as the conference itself have also begun to prepare researchers to target research agendas and themes, thus preparing them for recognizing and applying insights from future research.

# II. A RESEARCH AGENDA FOR STRENGTHENING THE PRACTICES OF MATHEMATICS EDUCATION

## SUMMARY RECOMMENDATIONS ON THEMES FOR A RESEARCH AGENDA

Mathematics education in America has been mainly understood as induction into a formal, technical discipline. The prevailing view of knowing and learning mathematics considers it as a body of abstract concepts and principles that support and generate powerful procedures for reasoning analytically both within the domain of mathematics and, at least potentially, in virtually all domains in which people need to think and understand. This view has the status of a folk theory—probably because it is consistent with almost everyone's experience of learning mathematics in American schools—and is also supported by a considerable body of research in behaviorist and cognitive psychology.

An alternative view of knowing and learning mathematics has emerged in the past 20 years that is supported by research about: children's intuitive understanding of mathematical concepts in mathematics education research; the development and study of classroom practices in which students contribute and advance their own and each others' conceptual understanding (e.g., Ball, Cobb, Lampert); the actual use of quantitative thinking and technical representations in practices of many professional workers (e.g., Hall, Goodwin & Goodwin, Ochs, Suchman & Trigg), and the quantitative reasoning in everyday activities of ordinary people (e.g., Lave, Nunes, Saxe). This alternative view considers mathematics (and other domains of conceptual meaning and representational technology) primarily as a resource for reasoning and communication in activity. In this view, knowing mathematics is sustained participation in practices in which mathematical contents (concepts, principles, and methods) are significant, and learning mathematics is becoming more effective in such practices. This view is also supported by recent analyses in epistemology that considers research and scholarship in mathematics (or science, history, or whatever) as a domain of practice in which researchers, teachers, and students participate, developing the body of knowledge, theory, and methods of inquiry as a socially organized enterprise (e.g., Kitcher, Putnam, Tymoszko).

This alternative is counterintuitive, as scientific alternatives to common sense often are. Like many scientific "revolutions," this theoretical development requires some basic concepts to switch places in an explanatory scheme. More specifically, the switch requires replacing an egocentric view with a view in which humans are considered primarily as participants in a larger system. It also requires replacing a concept of knowledge as a kind of structure with one of knowing as a kind of process. Both of these kinds of switches are known to be hard in both the histories of science and its present-day teaching.

This practice-based view of knowing and learning mathematics is an important aspect of the efforts to reform mathematics education. The NCTM *Standards,* for example, stated, "Three features of mathematics are embedded in the Standards. First, 'knowing' mathematics is 'doing' mathematics." The idea informed the development of standards-based curriculum funded by the NSF, including the MMAP curriculum that was discussed at the conference. It also has informed several design experiments, including the work of Cobb and his associates and others, which were also discussed at the conference.

The research results and developmental work reported and discussed at the conference indicate that the field is on the threshold of major advances both in scientific knowledge and theory about mathematics learning and teaching, in the design of methods of teacher education, and in the development of learning resources that could bring about fundamental improvements in school mathematics teaching and learning. The conference confirms the conclusions and recommendations of the PCAST committee on the use of technology to strengthen K-12 education in the United States, which characterized constructivist ideas as a "rich source of plausible and theoretically compelling hypotheses," and called for very substantial expansion of research resources to further develop and evaluate the empirical base and technological resources to support this approach to education in American schools.

The overall recommendation of the conference was for the National Science Foundation to consider funding a major program of studies related to how people learn and use mathematics throughout the life span. The group urges an increase in support for research that conceptualizes mathematics knowing and mathematics learning as practice, since these perspectives will be useful in bridging the gaps among research in different settings and continuing to develop educational materials and methods that will support fundamental improvements in mathematics teaching and learning.

The goal of such a research program would be to create a multidisciplinary mapping of the terrain of mathematical practices across the life span. Researchers would find complementary research questions, goals, activities and methods. Mathematical practice is known as different things in its different settings across the life span, and it is important to know and communicate what those are.

As part of the work, more research would be established on long-term trajectories into mathematical professions. Researchers would attempt to describe areas of exclusion and how people get in and out of the pipeline. The goal would be to align school math so it could be more than just one static aspect of the pipeline. The studies and projects would aim to strengthen connections between local communities and school math: Prototype projects for this kind of work currently include the Funds of Knowledge work at the University of Arizona and the Yupik math project in Alaska.

Some studies should be longitudinal and should examine trajectories of students coming out of reform curriculum K-12.

The studies should examine the role of technology to the study of mathematics considered broadly.

There should be a keen interest in establishing and studying projects that bring non-school mathematical practices to the mathematics classroom.

The mathematics learning research community should be broadened to include researchers who analyze practices in other settings (families, communities, and work places) who are interested in turning their attentions to mathematics.

Stakeholder communities are essential to the research that needs to be done.

**DISCUSSION**

The reports and discussions at the conference were concerned with an agenda of research and development in mathematics education that conceptualizes knowing and learning mathematics as *practices.* Recent findings of over twenty ongoing projects were presented and discussed, and participants addressed questions both about the current state of scientific and technical knowledge and understanding on this topic and about the prospective developments that can be expected from pursuing the topic energetically in the immediate future.

Research and development in mathematics education have entered a new phase in their continuing progress of improving the resources for successful teaching and learning of mathematics and of advancing the scientific knowledge base of this enterprise. During the years immediately following World War II, research and development focused on the cumulative learning of mathematics as the *acquisition of skill.* It was recognized that this was too mechanical an approach, and in the 1950s and 1960s, efforts were focused on developing curricula that emphasized *conceptual understanding* of mathematical principles that underlie procedures. Efforts to develop and implement mathematics curricula focused on conceptual understanding raised fundamental issues about the nature of learning with understanding, which were pursued in research. These efforts led to an understanding of the need to focus on *mathematical activities,* such as reasoning and communication, in students' learning of mathematics, expressed in the 1989 *Standards,* published by NCTM.

Efforts to develop curricula and collaborative work with teachers around the ideas of mathematics learning in the *Standards* have shown that successful teaching and learning requires that social arrangements of classroom activities must be organized differently from the didactic participation structures to which most teachers and students are accustomed. Research in work settings where workers engage in mathematical reasoning has also shown the importance of the social arrangements of activity in supporting successful use of mathematical knowing and understanding in work. This has led to an emphasis in the development of curriculum, programs of professional development, and research focused on the nature of social practices that support meaningful mathematical learning through productive participation in authentic mathematical activities.

The research and development efforts on this agenda are fundamentally multidisciplinary, drawing on resources from the social sciences, the cognitive sciences, and educational studies. Progress is being made toward combining the understanding of social interaction with programs in which students learn the content of subject-matter domains through significant participation in classroom discussions and project activity. A major theme in the research and development that is occurring is identification and understanding of differences in perspectives on what mathematics is, in different settings of social activity and in different cultural milieus. Significant progress is also being made in understanding the requirements and opportunities of teaching students whose linguistic and cultural backgrounds are different from those of the mainstream American culture for whom the patterns of discourse and activity of school tend to be reasonably well matched.

## STUDIES AND DEVELOPMENT OF PRACTICES IN SCHOOL

Much recent research and development activity in mathematics education has been directed toward understanding and increasing educational capabilities by changing the social organization of teaching and learning activities. The importance of a participation structure for learning in which students are expected to contribute their ideas and understanding (rather than simply receive authoritative knowledge) has been emphasized in the results of many studies, including studies by Lampert and Ball of their own teaching, and of Bransford and his associates, of Cobb and his associates, and of the IRL MMAP curriculum project, who have designed and studied mathematics learning environments in which students engage in complex problem solving, construction and interpretation of representations, and discussions in which they propose and criticize mathematical explanations. This corpus of work on mathematics learning in schools identifies social, cognitive, and organizational issues affecting mathematics learning. The group agreed that continued research is needed into best practices in K-12 mathematics. New work should build on, and expand, current and previous work. Conference researchers agreed that there are many versions of school math. It would be helpful to understand the full spectrum of classroom practices, yet this would demand a massively comprehensive set of studies to produce the needed knowledge.

In working to envision a feasible approach to a comprehensive study of mathematics classroom practices, researchers thought a series of studies of "best and most successful practices" could be most productive. In a similar vein, the group recognized the success of combining research and reform efforts. Research is able to yield important learning in the field, for example, while addressing support of teachers' instructional strategies and practices, the creation and implementation of standards-based curriculum materials and assessment practices and tools.

Some recommendations for such an agenda include:

- Study classrooms and schools that are having success. Focus on the practices in those environments, aiming toward analyses that show how social interactions are organized to promote students' progress in learning concepts and methods of subject-matter domains.

- Take *community as* a primary level of research and analysis. Examine what happens in classrooms, but also look at how policy and community level processes and decisions support learning and

implementing. This would also enable us to better understand and change assumptions about the relationship between policy and classroom practice.

- Look at schools that are a representative sample of US schools. Criteria for selection of schools studied would need to be clearly delineated.

- Study out of school math settings and work places that serve K-12 students, especially to inform the design and development of mathematics curricula that support school learning that has continuity with students' out-of-school activity.

Things to understand:

- Continue to analyze, describe, and catalogue the cognitive processes in students' school mathematics activity to advance the scientific knowledge base of mathematics education and inform continued improvement of curriculum, teaching practice, and teacher education.

- See if there are common successful teaching and learning practices that span all of the contexts.

- Begin to understand how policies and curriculum and technological resources function as constraints and are adapted in locally constituted practices.

- Research on assessment practices needs to inform decisions about what we think is important to be measured in math learning and how we should measure it.

- Understand good teaching and professional practices in these situations. Learn how teachers form communities for learning and growing. What is the process of change a teacher goes through over time? Understand how teachers communicate and mediate experience for various stakeholders in the mathematics education process.

- Support studies that combine empirical and analytical methods across disciplines. Progress toward both the scientific and practical goals of research and development require multidisciplinary resources, and collaborative studies in which researchers can share research methods and analyses are of special importance.

## MATHEMATICAL PRACTICES OUT OF SCHOOL

The review of prior work on mathematical practices in out-of-school settings is uncovering many mathematical processes people accomplish in everyday activities. This work has taken several directions. We have learned some about gaps between learning mathematics in and out of school (Carraher, Carraher, and Schlieman, D'Ambrosio, Lave, Lester, Nunez, Saxe). When researchers look at how people use mathematics in everyday settings such as shopping or in street businesses, they see they are quite adept at problem-solving and with the mathematical demands of the settings in which they act. This body of work has led educators to consider more contextualized situations (or problem situations that parallel real–world situations, materials, and people) to engage students, and to increase higher level thinking and students' ownership of problems (Brown, et al., Resnick, Romberg, Schoenfeld). Current curriculum guidelines and standards for mathematics (such as the NCTM Standards, the Standards 2000 draft, and many state frameworks call for engaging students in problem-solving approaches and the application of mathematics to the "real–world." IRL's work in middle school mathematics, focused on placing of students in "real-world" design problems that require the use of mathematics concepts as tools. By experiencing the utility of applying these concepts as tools to find solutions to problems they find relevant, students have a chance to appreciate the power of the mathematics and its importance to their future work. Similar research on school mathematics is based on similar approaches (Bransford et. al, Leinhardt).

Several researchers studying out of school settings have focussed on the role of mathematics. These works have included research cultural approaches to mathematics, mathematics activity in the work place, and studies of highly demanding technical and scientific work settings where mathematical work is central and overtly dominates job entry requirements and everyday work experience. Eglash has looked at fractal patterns as a basic pattern in African villages, Lipka has spent over a decade collaborating with Yupik villagers to understand Yupik ways of learning and knowing mathematics. Researchers at the University of Arizona have studied the "funds of knowledge" in minority families that can be drawn on for increasing school success. In the work place, recent work by Linde and her collaborators examines the high need for mathematical literacy needed by both sales agent and customer in life insurance transactions. Researchers studying how mathematics is practiced in scientific, technical, and engineering settings (Hall, Fujimura, Osgood) helps to delineate the demands for mathematics learning and performance in the fastest growing work environments.

The researchers would like to see coordinated work that reveals how mathematical knowledge develops in such practices (take on issues of social organization, valued artifacts, norms & values, patterns of interaction, access).

Some areas that would be good areas for future study include:

- Selling magazines for prizes (Math topics: rational number, multi-digit arithmetic, and decimal fractions)
- Gambling (Math topics: probability)
- Games (Math topics: Are their analogs for mathematical proofs? Justifying move or action; coordinating claims and evidence in relation to rules of play; Kids are jury of their peers inside specified of rules – warrants & backings)
- Internet communities of kids engaged with hacking (math Topics: discrete math, logic, and algebra)
- Home Hobbies/Home Businesses
- Household budget and management (household budgets, saving, buying insurance or making investments).

The overall goals of the research program would make it possible to sample how children and adults transact mathematically in communities. New studies would seek to document existence of practices, frequency of occurrence, math density of practices, types of mathematical domains that emerge in various communities of practice. The analyses of the research would focus on selecting and analyzing mathematically dense practices to better understand how they are organized socially, structurally, and cognitively for people.

## MATHEMATICAL PRACTICES IN THE WORK PLACE

Work places are varied and, in some cases, intense settings for mathematical practices, and therefore, a special out-of-school arena for study. The NSF needs to focus research around math in the workplace, and training in the workplace. Some questions that might drive such an agenda are: What math gets used in various kinds of workplaces? What inscriptions and artifacts are used? How is technology affecting mathematics in different work settings? How is mathematics negotiated in employee training?

Some prior work has been done on mathematics in work settings, but the work is in the nascent stage (Hall at Berkeley, IRL). In some of the previous studies, mathematics practices were studied directly, while in the majority of studies the focus was on cognitive processes more generally with mathematics practices emerging incidentally. Settings studied include architecture, biology, insurance, baggage handlers, dairy workers, and carpenters.

The group had the notion that the NSF might be able to collaborate with the Dept. of Labor to develop joint programs for this aspect of the transition to work research agenda.

## III. BRINGING MATHEMATICS IN SCHOOL, OUT-OF-SCHOOL AND WORK TOGETHER

The overall recommendation of the conference is for the National Science Foundation to consider funding a coordinated program of studies of how people learn and use mathematics throughout the life span. The group urges an increase in support for research that conceptualizes mathematics knowing and mathematics learning as practice, since this perspective has important scientific potential and will be useful in bridging the gaps among research in schools, communities and work settings.

The goal is to establish a multidisciplinary mapping of the terrain of mathematical practices across the life span. The overall project would uncover and describe mathematical practices in different settings across the life span, and *start the important work of strengthening connections across the settings and practices*.

The research would aim to understand more about long-term trajectories into mathematics using work settings and begin to strengthen alignments of local communities, school and work place mathematics practices.

The mathematics education research community will have to expand to meet these goals. Researchers who analyze practices in other settings (families, communities, and work places) will need to turn their attentions to mathematics.

## IV. STUDIES AND DEVELOPMENT OF RESOURCES FOR LEARNING IN MATHEMATICAL PRACTICE

The research and development efforts the group discussed provide a strong basis for continued efforts, as the previous sections of this report have shown. As these efforts continue, it is important for several general considerations to be kept in mind.

First, it is crucial for programs of research and development to accumulate—that is, to develop bodies of research findings and analyses of practices that add to the knowledge base available for educational policy and successful practice. It is also crucial that the accumulated body of scientific knowledge and understanding should make information available to practitioners and policy officials for use in their decisions and planning. One promising direction is in the kind of site-based research and development, recently recommended by the National Academy of Education study for the National Educational Research Policy and Priorities Board, in which researchers collaborate with practitioners in working on improving educational effectiveness in a local setting, and carry out empirical studies and theoretical analyses that contribute to understanding fundamental processes of learning, social interaction, and organizational behavior. Such studies provide settings that are especially appropriate for the kinds of interdisciplinary efforts that are needed to further the agenda of studying learning and teaching mathematics as practices.

Efforts for research and school improvement need to include work to develop and understand organizational structures that support teaching and professional development. It is crucial that we understand the consequences of inequities in financing, teacher turnover, and other resources that are distributed unevenly among our schools, and develop policies that reflect America's commitment to providing equal educational opportunity for all of our children.

## V. PROPOSED CONTINUED INTERACTIONS FOR THE CONFERENCE RESEARCHERS

The researchers gained a great deal of understanding from sharing research work across school, informal and work settings. The researchers discussed patterns of teaching and learning in each setting, as well as methods, analyses and findings. This was an extremely worthwhile activity for the researchers, and the level of synthesis of ideas and methods was high. The participating researchers thought the momentum gained should not be lost. A next conference would bring together this group, with a few additional researchers, to begin examining data from different math using and learning settings. This would be a first step in seeing whether currently available work, which was conducted with disparate goals, uncommon research questions, methods and analytic techniques could somehow be used to answer the specific goals we have outlined.

More specifically, the researchers would like to choose data from five different mathematics settings across the life span and begin the process of determining whether alignment is possible.

The activity of a second meeting would move towards shared in depth analyses, yet build on the insights and momentum developed in the first meeting.

Some of what the group imagined would be worthwhile activity:

- Looking at already collected data together and seeing if we can make connections among them
- Figuring out the validity of the work we do. How can this agenda progress what are the standards for the validity of this work?
- Developing the standards for doing cross-disciplinary, cross-setting work in accountable ways.
- Figuring out how emerging technologies can be used to improve research data collection, analyses and dissemination.
- Developing common ways for conveying the results of research. People noted how the phonics proponents have gotten their message out loud and clear and how math researchers needs to be able to communicate as succinctly and effectively.
- Judging whether it is possible and/or economical in terms of dollars, time, and effort to re-purpose already collected data for these purposes or whether new research is essential.
- Beginning to train subsequent generations of researchers to do this work.
- Start developing proposals for the management of comprehensive research programs. If you have national agenda, how do you manage the projects, keep research on target, keep things on schedule and keep goals and mission aligned?

A second proposed activity to maintain interest and clarify the importance of a math across the life span research agenda is to produce a white paper or volume that describes the force of mathematics as a powerful tool of society. Researchers would focus on how mathematical power is very tied to opportunity and equity in school, work and life our democratic society

This issue of "mathematics as power" came up repeatedly at the conference and was recognized as having immense equity implications. Conference participants asked: What role does mathematics play in giving/denying access to learning and work opportunities? What does it mean to have more kids achieve in mathematics when math in both school and the real world is a moving target? Future activities might take up the topic of what counts as legitimate knowledge in mathematics in different settings at different times and

how what is legitimate contribute to success and failure throughout the life span? We cited many examples such as the following that are informative of the issues educators and math learners have to face. Children do sophisticated mathematics "off the record," but their achievements are not acknowledged by the schools. In Alaska, during an interaction between scientists and native leaders, scientists' knowledge of weather was considered legitimate while elders knowledge was not, even when the elders were better at predication. In a work place, workers used complicated mathematics and even taught it to their clients on the job, but failed a simple school-like arithmetic test during training.

The group has in mind that it is of utmost importance for the Foundation to award small grants for collaborative syntheses of the literature or a small conference around how mathematics power functions in the culture and throughout the life span.